Chapter 39: Particles Behaving as Waves

Group Members:

1. An electron is moving with a speed of $8.00\times10^{6}\ m/s$.

   (Useful data: $h=6.63\times10^{-34}\,J\cdot s;\ m_e=9.11\times10^{-31}\,kg;\ m_p=1.67\times10^{-27}\,kg$)

   a. What is the electron's de Broglie wavelength (in *nm*)?

   $$\lambda=\frac{h}{p}=\frac{h}{m_e v}=\frac{6.63\times10^{-34}\,J\cdot s}{\left(9.11\times10^{-31}\,kg\right)\left(8.00\times10^{6}\ m/s\right)}=0.0910\,nm$$

   b. What is the speed of a proton that has the same de Broglie wavelength as this electron?

   For a proton with the same de Broglie wavelength as the electron, it will have the same momentum as the electron,

   $$p_p=p_e=\left(9.11\times10^{-31}\,kg\right)\left(8.00\times10^{6}\ m/s\right)=7.288\times10^{-28}\,kg\cdot m/s$$

   So, $p_p=m_p v=7.288\times10^{-24}\,kg\cdot m/s$ and solving for v, we have,

   $$v=\frac{p_p}{m_p}=\frac{7.288\times10^{-24}\,kg\cdot m/s}{1.67\times10^{-27}\,kg}=4.36\times10^{3}\ m/s$$

   c. Now, what will be the de Broglie wavelength of a cannon ball with mass $m_c=180.kg$ moving at the speed given in part b?

   The cannon ball will have a momentum of

   $$p_c=m_c v=180kg\left(4.3641\times10^{3}\ m/s\right)=7.855\times10^{5}\,kg\cdot m/s$$

And, it will have a de Broglie wavelength of,

$$\lambda = \frac{h}{p_c} = \frac{6.63\times10^{-34}\,J\cdot s}{7.855\times10^{5}\,kg\cdot m/s} = 8.44\times10^{-40}\,m$$

2. The energy level of a hydrogen atom is given by $E_n = \frac{-13.6eV}{n^2}$. An electron is initially at an excited state ($n = 3$). When the atom relaxes back to its ground state, a photon is released. (Useful data: $h = 6.63\times10^{-34} J\cdot s$; $1eV = 1.60\times10^{-19} J$; $hc = 1241 eV\cdot nm$)

   a. Determine the energy of that photon.

$$E_{photon} = \Delta E = E_3 - E_1 = -13.6eV\left(\frac{1}{3^2} - \frac{1}{1^2}\right)$$
$$= -13.6eV(-0.888) = 12.089eV = 12.1eV$$

   b. Determine the wavelength of that photon.

Then, using the relation $E = hf = \frac{hc}{\lambda}$, we can calculate the wavelength of the light,

$$\lambda = \frac{hc}{E_{photon}} = \frac{1242eV\cdot nm}{12.088eV} = 102.7nm = 103nm$$

   c. An electron at the ground state of the hydrogen atom absorbs a photon with a wavelength of $90.0nm$, will the energy absorbed sufficient to ionize the hydrogen atom?

The photon will have energy given by,

$$E_{photon} = \frac{hc}{\lambda} = \frac{1242eV\cdot nm}{90nm} = 13.8eV$$

For the electron at ground state ($n = 1$) to be ionized, it needs to reach $n = \infty$ and the minimum energy needed is,

$$\Delta E = E_f - E_i = 13.6eV\left(-\frac{1}{\infty} + \frac{1}{1}\right) = 13.6eV$$

Since $13.8eV > 13.6eV$, the hydrogen atom will be ionized.

3. A $10.0g$ marble is gently placed on a large tabletop.

   (Useful data: $h = 6.63\times10^{-34} J\cdot s$)

   a. If the table is $1.75m$ wide along the x - direction, what is the maximum uncertainty in the x position of the marble?

      The marble can be anywhere on the tabletop so the maximum uncertainty in x is,

      $\Delta x = 1.75m$

   b. According to the Heisenberg uncertainty principle, what is the minimum uncertainty in the velocity of the marble in the x-direction?

      Heisenberg uncertainty principle requires that $\Delta p_x \Delta x > \frac{\hbar}{2}$, so solving for Δp_x, we have,

      $$\Delta p_x > \frac{\hbar}{2\Delta x} = \frac{6.63\times10^{-34} J\cdot s/2\pi}{2(1.75m)} = 3.015\times10^{-35} kg\cdot m/s$$

      Assuming the marble to have a minimum momentum in the x-direction given by its uncertainty calculated in b), we can calculate its minimum velocity to be

      $$v_{x,\min} = \frac{\Delta p_x}{m} = \frac{3.015\times10^{-35} kg\cdot m/s}{1.00\times10^{-2} kg} = 3.015\times10^{-33} m/s$$

   c. Considering the x-direction only, what is the longest time the marble could remain on the table?

      Now, assuming the worst case scenario in which the marble needs to traverse the entire wide $1.75m$ of the table, we can calculate the time needed for it to do so be,

      $$t = \frac{1.75m}{3.015\times10^{-33} m/s} = 5.80\times10^{32} s = 1.84\times10^{25} years$$